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i. Let $z=2r$. $x+2y=6(n-r)$. y may have any value from 0 to $3(n-r)$.

\therefore There are $3(n-r)+1$ solutions when z is $2r$.

\therefore Total number of solutions for z even is

$$\begin{aligned}\sum_{r=0}^{r=n} [3(n-r)+1] &= (3n+1)(n+1) - \frac{3n(n+1)}{2} \\ &= \frac{n+1}{2} (6n+2-3n) = \frac{(n+1)(3n+2)}{2}.\end{aligned}$$

ii. Let $z=2r+1$. $x+2y=6(n-r)-3$. y may have any value from 0 to $3(n-r)-2$.

$\therefore 3(n-r)-2+1=3(n-r)-1$ solutions.

\therefore Total number of solutions when z is odd:

$$\sum_{r=0}^{r=n-1} [3(n-r)-1] = (3n-1)n - \frac{3n(n-1)}{2} = \frac{n}{2} (6n-2-3n+3) = \frac{n}{2} (3n+1).$$

\therefore The total number of solutions

$$= \frac{(n+1)(3n+2)}{2} + \frac{n(3n+1)}{2} = \frac{3n^2+5n+2+3n^2+n}{2} = 3n^2+3n+1.$$

Also solved by H. Prime, J. Scheffer, H. C. Feemster, and A. M. Harding.

363. Proposed by E. B. ESCOTT, Ann Arbor, Mich.

(a) If a and n be positive integers, the integral part of $[a+\sqrt{(a^2-1)}]^n$ is odd.

(b) If a and n be positive integers, the integral part of $[\sqrt{(a^2+1)}+a]^n$ is odd when n is even and even when n is odd. [From Todhunter's *Algebra*, p. 353].

I. Solution by the PROPOSER.

Proof. (a) Let $[a+\sqrt{(a^2-1)}]^n = P+Q\sqrt{(a^2-1)} = m$.

Then $[a-\sqrt{(a^2-1)}]^n = P-Q\sqrt{(a^2-1)} = 1/[a+\sqrt{(a^2-1)}]^n$.

$\therefore 0 < P-Q\sqrt{(a^2-1)} < 1$.

Adding m to each member of the inequality $m < 2P < m+1$.

Therefore, the integral part of m is odd.

(b) Let $[\sqrt{(a^2+1)}+a]^n = R+S\sqrt{(a^2+1)} = k$.

Then $[-\sqrt{(a^2+1)}+a]^n = R-S\sqrt{(a^2+1)} = \left(\frac{-1}{\sqrt{(a^2+1)}+a}\right)^n$.

If n is even, $0 < R-S\sqrt{(a^2+1)} < 1$. Adding k , $k < 2R < k+1$. Whence the integral part of k is odd.

If n is odd, $-1 < R - S\sqrt{a^2+1} < 0$. Adding k , $k-1 < 2R < k$. Whence the integral part of k is even.

Also solved similarly by S. Lefschetz.

II. Solution by W. J. GREENSTREET, M. A., Editor of the Mathematical Gazette, Burghfield, England, and J. SCHEFFER, A. M., Hagerstown, Maryland.

Let I be the integral part of $[a + \sqrt{a^2-1}]^n$.

Then $I + F = [a + \sqrt{a^2-1}]^n$, where F is a proper fraction,

$$= a^n + n a^{n-1} \sqrt{a^2-1} + \frac{n(n-1)}{2!} (a^2-1) + \frac{n(n-1)(n-2)}{3!} (a^2-1)^{\frac{3}{2}} + \dots$$

Also, $a > \sqrt{a^2-1}$; $\therefore a - \sqrt{a^2-1} < 1$, and $[a - \sqrt{a^2-1}]^n < 1$.

Let $F' = [a - \sqrt{a^2-1}]^n = a^n - n a^{n-1} \sqrt{a^2-1}$

$$+ \frac{n(n-1)}{2!} (a^2-1) - \frac{n(n-1)(n-2)}{3!} (a^2-1)^{\frac{3}{2}} + \dots$$

$$\therefore I + F + F' = 2 \left(a^n + \frac{n(n-1)}{2!} (a^2-1) + \dots \right) = 2p \text{ (say)}.$$

Hence, $F + F' = 1$, and $I = 2p - 1$ and is odd.

Similarly, $I + F = [\sqrt{a^2+1} + a]^n = (a^2+1)^{\frac{1}{2}n} + n(a^2+1)^{\frac{1}{2}(n-1)} a$

$$+ \frac{n(n-1)}{2!} (a^2+1)^{\frac{1}{2}(n-1)} a^2 + \dots$$

$$F' = [\sqrt{a^2+1} - a]^n = (a^2+1)^{\frac{1}{2}n} - n(a^2+1)^{\frac{1}{2}(n-1)} a + \dots$$

(i) n odd. $I + F - F' = 2[n(a^2+1)^{\frac{1}{2}(n-1)} a + \dots] = 2p$ (say) = an even integer.

$\therefore F - F' = 0$, and I is an even integer.

(ii) n even. $I + F + F' = 2[(a^2+1)^{\frac{1}{2}n} + \frac{n(n-1)}{2!} (a^2+1)^{\frac{1}{2}(n-2)} a^2 + \dots] = \text{an even integer.}$

$\therefore F + F' = 1$, and I is an odd integer.